EPISTEMOLOGY AND THEORY OF MACHINE LEARNING

EXERCISE SET #6

(1) Let $\mathcal{H}_n$ for $n \in \mathbb{N}$ be the hypothesis class of unions of n intervals,

$$\mathcal{H}_n = \{h_{a_1,b_1,\ldots,a_n,b_n} : (\forall i \leq n)(a_i \leq b_i)\}$$

where

$$h_{a_1,b_1,\ldots,a_n,b_n}(x) = 1 \text{ iff } x \in \cup_{i \leq n}[a_i, b_i].$$

Show that the hypothesis class $\mathcal{H} := \cup_n \mathcal{H}_n$ of all unions of intervals is *not* PAC learnable, but *is* nonuniformly learnable.

(2) Let $\mathcal{H}^{\text{all}}$ be the class of *all* hypotheses $h : [0, 1] \to \{0, 1\}$. Show that $\mathcal{H}^{\text{all}}$ is not even nonuniformly learnable.

Hint: Show that if $\mathcal{H}$ is a class that shatters an infinite set K, then for each possible decomposition $\mathcal{H} = \cup_n \mathcal{H}_n$ there must be an $\mathcal{H}_n$ with infinite VC dimension. Specifically, show, towards a contradiction, that otherwise we could do the following. First, construct a sequence of disjoint subsets $K_n \subseteq K$ such that $|K_n| > \text{VCdim}(\mathcal{H}_n)$. Further, pick functions $f_n : K_n \to \{0, 1\}$ which are not in the restriction of $\mathcal{H}_n$ to K_n, and combine those into a function f which is in $\mathcal{H}$ but not in $\cup_n \mathcal{H}_n$.