PHILOSOPHICAL INTRODUCTION
TO COMPUTABILITY AND RANDOMNESS
ASSIGNMENT #1

Deadline: **Friday June 7**, before class. Strive to answer concisely yet informatively. Aim for about half a page per question; certainly not more than one page.

(1) Freudenthal remarked that

> It may be taken for granted that any attempt at defining disorder in a formal way will lead to a contradiction. This does not mean that the notion of disorder is contradictory. It is so, however, as soon as I try to formalize it.

Explain in what sense the Berry paradox is an instance of this statement. How may the contradiction be avoided by bringing in the notion of computability?

(2) Show that:
   (a) $C(\pi^n) \leq^+ C(0^n)$, where π^n denotes the first n bits of the binary expansion of $\pi = 3.1415\ldots$, and 0^n denotes the sequence of n 0's.
   (b) $C(0^n) \leq^+ \lceil \log_2 n \rceil$, with $\log_2$ the binary logarithm and $\lceil \cdot \rceil$ the ceiling function, e.g., $\lceil 1.4 \rceil = 2$.

(3) Dasgupta (2011, 687f) proclaims that with the definition of Kolmogorov complexity, "the Laplace program is realized." Do you agree, and why (not)? Be sure to mention either the invariance or the incomputability of Kolmogorov complexity.